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Quantum Field Formalism for the Electromagnetic Interaction of Composite Particles in a Nonrelativistic Gauge Model

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A classical nonrelativistic $U(1) \times U(1)$ gauge field model for the electromagnetic interaction of composite particles is proposed and the quantum formalism is constructed. This gauge model containing a Chern–Simons $U(1)$ field and the electromagnetic $U(1)$ field can be coupled to both a bosonic or a fermionic matter field. We explicitly consider the second case, a composite fermion system in the presence of an electromagnetic field, and we carry out the canonical quantization by the Dirac method. The path integral approach is developed and the Feynman rules are established. A simplified model is considered. As an alternative path integral method, the BRST formalism for this gauge model is also treated.

1. INTRODUCTION

We start with a brief consideration of composite bosons (CBs) (Girvin and MacDonald, 1987; Lee and Fisher, 1989; Lee and Zhang, 1991; Read, 1989; Zhang, 1992; Zhang *et al.*, 1989) and composite fermions (CFs) (Jain, 1989a, 1990; Jain and Kamilla, 1998) in the context of the quantum Hall effect (QHE) and its integer (IQHE) and fractional (FQHE) versions.

Even though the experimental observations of the IQHE and the FQHE are essentially identical, except for the value of the quantized Hall resistance, the possibility of a relationship between them had not been contemplated, mainly because the FQHE was believed to be fully explainable within the lowest Landau level (LL) whereas the IQHE clearly required the higher LLs.

Furthermore, while the IQHE is thought of essentially as a noninteracting electron phenomenon (Laughlin, 1981; Halperin, 1982), the FQHE is believed to arise from the condensation of the two-dimensional electrons into a "new collective state of matter" as a result of interelectron interactions (Laughlin, 1983).

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One of the models used to explain the FQHE is the so-called quasi-particle hierarchy (QPH) approach, started by Laughlin (1983), who proposed an ansatz wave function to describe the correlated electron low-energy liquid at $\nu = 1/(2n + 1) = 1/3,\ 1/5,\ 1/7, \ldots$, where ν is the filling factor and n is an integer. It was compared by Laughlin (1983) and other investigators (Fano *et al.*, 1986; Haldane and Reyazi, 1985) with the exact numerical ground-state wave function of few electron systems and was found to be extremely accurate.

Laughlin also constructed wave functions for the quasi-particle excitations and made compelling arguments that there was a finite gap in the excitation spectrum, resulting in FQHE with fractions $f = 1/(2n + 1)$ ($f = h/(e^2 R_{\mathrm{H}})$), where h is Planck's constant, e is the fundamental charge, and R_{H} is the Hall resistance on the plateaus).

Later on, Haldane (1983) and Halperin (1984) proposed iterative hierarchical methods that extend the QPH approach to the other odd-denominator fractions. These methods suppose that "daughter" states are obtained at each step, when the quasi-particles of a "parent" state condense themselves into a Laughlin-like state. So, $1/3$ generated daughters at $2/5$ and $2/7$, which in its turn produced daughters at $5/17$, $3/11$, $5/13$, and $3/7$, and so on.

This last scheme was not completely convincing (Jain, 1992) because it should have explained all fractions on a more or less equal footing.

This pointed out that the physics of the Laughlin wave function was itself not fully understood. There were several attempts to elucidate the relevant correlations in the Laughlin wave function, as we will show in the following.

It is important to develop an effective-field-theory model of the FQHE analogous to the Landau–Ginzburg theory of superconductivity. In this sense, Girvin and MacDonald (1987) proposed a field-theory model containing a complex scalar field coupled to a gauge vector field with a Chern–Simons (CS) action. This model exhibits vortex solutions with finite energy and fractional charge that can be identified with Laughlin's quasi-particles and quasi-holes. The amplitude fluctuations of the scalar field are massive and are identified with the density-fluctuation modes of the single-mode approximation (Girvin *et al.*, 1985, 1986). So, the CB theory arose.

Zhang *et al.* (1989) and Read (1989) proposed a mean field theory in which the Laughlin wave function was viewed as a Bose condensate.

These theories, however, did not give a better explanation of the other odd-denominator fractions.

CFs were introduced by Jain (1989a, 1990; Jain and Kamilla, 1998). The motivation was to provide a unified description of the IQHE and the FQHE and a source of excellent trial wave functions for the series $\nu = p/(2np \pm 1)$, where n and p are integers.

The CS approach to two-dimensional electrons in a strong perpendicular magnetic field at or close to half filling of the lowest LL ($\nu = 1/2$), as pioneered by

Halperin *et al.* (1993), is well supported by experiments (Willett *et al.*, 1990, 1993). These investigators identified the low-temperature phase of fully spin-polarized electrons at half filling as a Fermi liquid of CFs, each consisting of an electron with two attached flux quanta in a zero magnetic field (Jain, 1989b; Lopez and Fradkin, 1991).

They pointed out that their CF formalism easily generalizes to other filling factors and to higher LLs, where, if applicable, it describes a Fermi liquid phase for spin-polarized electrons at all even-denominator filling factors. In this case, the CFs can be viewed as electrons carrying an even number of flux quanta.

In the present paper, we start by considering a composite particle system coupled to two $U(1)$ gauge fields, a CS one a_μ (Zhang *et al.*, 1989; Wilczek, 1982; Arovas *et al.*, 1985), and the other the electromagnetic field A_μ. The purpose of our paper is to analyze this system in a particular classical nonrelativistic $U(1) \times U(1)$ gauge field model and to study this model from the quantum point of view.

On the other hand, the classical and quantum CS theories in two spatial dimensions coupled to different types of matter field have been known since a long time ago (Deser *et al.*, 1982a,b, 1988; Dunne *et al.*, 1989; Jackiw and Templeton, 1981; Matsuyama, 1990a,b; Avdeev *et al.*, 1992; Lin and Ni, 1990; Odintsov, 1992); therefore, this fact will be profitable in all the constructive procedure.

The paper is organized as follows. In section 2, we present our gauge model from the classical point of view. Later on, we analyze the set of constraints and perform the canonical quantization of the model following the prescriptions of the Dirac formalism for constrained Hamiltonian systems. In section 3, by using the path integral method, we establish the Feynman rules of the model. Later, in section 4, we consider a reduced model related to a known one. Finally, in section 5, the BRST formalism is also treated.

2. CLASSICAL GAUGE MODEL: CANONICAL QUANTIZATION

We are going to consider a classical nonrelativistic field theory with $U(1) \times U(1)$ gauge symmetry for the electromagnetic interaction of composite particles in $(2+1)$ dimensions. In particular, we will analyze a CF system. We will assume that this system can be described by the following singular Lagrangian density:

$$\mathcal{L} = \mathcal{L}_{\mathrm{cf}}^{\mathrm{em}} + \mathcal{L}_{\mathrm{em}}, \tag{2.1}$$

where $\mathcal{L}_{\mathrm{cf}}^{\mathrm{em}}$ is written as

$$\mathcal{L}_{\mathrm{cf}}^{\mathrm{em}} = i\psi^\dagger \mathcal{D}_0 \psi + \frac{1}{2m_{\mathrm{b}}}\psi^\dagger \vec{\mathcal{D}}^2 \psi - \mu \psi^\dagger \psi + \frac{1}{4\pi\tilde{\phi}} \varepsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \tag{2.2a}$$

and $\mathcal{L}_{\text{em}}$ reads

$$\mathcal{L}_{\text{em}} = -\frac{1}{4} F_{\mu\nu}(A) F^{\mu\nu}(A). \tag{2.2b}$$

In Eqs. (2.2), the Greek indices take the values μ, ν, $\rho = 0, 1, 2$.

We employ units where $\hbar = c = 1$. The Minkowskian metric is $g_{\mu\nu} = \text{diag}\,(1, -1, -1)$ and $\varepsilon^{012} = \varepsilon^{12} = 1$.

In Eq. (2.2a), the covariant derivative, involving both the CS $U(1)$ gauge field a_μ and the electromagnetic $U(1)$ gauge field A_μ, is written as $\mathcal{D}_\mu = \partial_\mu - ia_\mu - ieA_\mu$ and we designate $\vec{\mathcal{D}}^2 = \mathcal{D}_1^2 + \mathcal{D}_2^2$. The matter field ψ is a charged spinorial field describing CFs. The electron charge is taken as $-e$. m_{b} is the band mass of the electrons. μ is the chemical potential for electrons. $\tilde{\phi}$ is the strength of the flux tube, in units of the flux quantum 2π. (The fictitious charge of each particle that interacts with the fictitious gauge field has been chosen to have unit strength.)

A CB system can be considered along the same lines, the only difference is that, in this case, the matter field is a charged scalar field.

By using the expression for the covariant derivative, we can rewrite Eq. (2.2a) as

$$\begin{aligned} \mathcal{L}_{\text{cf}}^{\text{em}} &= i\frac{\tau+1}{2}\psi^\dagger \partial_0 \psi + i\frac{\tau-1}{2}\partial_0 \psi^\dagger \psi + \psi^\dagger (a_0 + eA_0)\psi \\ &\quad + \frac{1}{2m_{\text{b}}}\psi^\dagger \vec{\mathcal{D}}^2 \psi - \mu\psi^\dagger\psi + \frac{1}{4\pi\tilde{\phi}}\varepsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho. \end{aligned} \tag{2.3}$$

In Eq. (2.3), the kinetic fermionic term is written in the general form by using the arbitrary parameter τ, which is the usual way to obtain symmetric expressions for the canonically conjugate momenta corresponding to the matter fields $\psi^\dagger$ and ψ (Sundermeyer, 1982).

The canonical quantization is carried out by using the Dirac algorithm (Sundermeyer, 1982; Dirac, 1964).

The momenta p^μ, P^μ, $\pi_\alpha^\dagger$, and π_α canonically conjugate to the independent field variables a_μ, A_μ, ψ_α, and $\psi_\alpha^\dagger$, respectively, remain given by

$$p^0 = 0, \tag{2.4a}$$

$$p^i = \frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij} a_j, \tag{2.4b}$$

$$P^0 = 0, \tag{2.4c}$$

$$P^i = F^{i0}(A), \tag{2.4d}$$

$$\pi_\alpha^\dagger = \frac{\partial \mathcal{L}_{\text{cf}}^{\text{em}}}{\partial \dot{\psi}_\alpha} = -i\,\frac{\tau+1}{2}\psi_\alpha^\dagger, \tag{2.4e}$$

$$\pi_\alpha = \frac{\partial \mathcal{L}_{\rm cf}^{\rm em}}{\partial \dot{\psi}_\alpha^\dagger} = i\frac{\tau - 1}{2}\psi_\alpha, \tag{2.4f}$$

where the Latin indices take the values $i, j = 1, 2$ and the Greek index takes the values $\alpha = 1, 2$.

As is usual, the nonvanishing fundamental equal-time ($x^0 = y^0$) Bose–Fermi brackets are those used for pairs of canonically conjugate variables and are given by

$$[a_\mu(x), p^\nu(y)]_- = \delta_\mu^\nu \delta(\vec{x} - \vec{y}), \tag{2.5a}$$

$$[A_\mu(x), P^\nu(y)]_- = \delta_\mu^\nu \delta(\vec{x} - \vec{y}), \tag{2.5b}$$

$$[\psi_\alpha(x), \pi_\beta^\dagger(y)]_+ = -\delta_{\alpha\beta}\delta(\vec{x} - \vec{y}), \tag{2.5c}$$

$$[\psi_\alpha^\dagger(x), \pi_\beta(y)]_+ = -\delta_{\alpha\beta}\delta(\vec{x} - \vec{y}). \tag{2.5d}$$

Here, we have used the notation $[.\,,.]_\mp$ to point out brackets between bosonic and fermionic Grassmannian variables, respectively.

Looking at Eqs. (2.4), it can be seen that the primary constraints are

$$\Phi_1^0 = p^0 \approx 0, \tag{2.6a}$$

$$\Phi_2^{0i} = p^i - \frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij}a_j \approx 0, \tag{2.6b}$$

$$\Phi_3^0 = P^0 \approx 0, \tag{2.6c}$$

$$\Omega_\alpha^\dagger = \pi_\alpha^\dagger + i\frac{\tau + 1}{2}\psi_\alpha^\dagger \approx 0, \tag{2.6d}$$

$$\Omega_\alpha = \pi_\alpha - i\frac{\tau - 1}{2}\psi_\alpha \approx 0, \tag{2.6e}$$

with $i, j = 1, 2$ and $\alpha = 1, 2$.

Now, the primary Hamiltonian $H_{\rm p} = \int d^2x\, \mathcal{H}_{\rm p}$ remains defined in terms of the following Hamiltonian density:

$$\mathcal{H}_{\rm p} = \mathcal{H}_{\rm c} + \lambda_1\Phi_1^0 + \lambda_{2i}\Phi_2^{0i} + \lambda_3\Phi_3^0 + \lambda_\alpha^\dagger\Omega_\alpha + \Omega_\alpha^\dagger\lambda_\alpha, \tag{2.7}$$

where $\lambda_1, \lambda_{2i}, i = 1, 2,$ and λ_3 are bosonic Lagrange multipliers and $\lambda_\alpha^\dagger$ and $\lambda_\alpha, \alpha = 1, 2$, are fermionic ones.

In Eq. (2.7), the functional $\mathcal{H}_{\rm c}$ is defined as usual by $\mathcal{H}_{\rm c} = \dot{a}_\mu p^\mu + \dot{A}_\mu P^\mu + \dot{\psi}\pi^\dagger + \dot{\psi}^\dagger\pi - \mathcal{L}$, which, after Eqs. (2.4) have been used, writes

$$\mathcal{H}_{\rm c} = -\frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij}a_0\partial_i a_j + \partial_i a_0 p^i + \frac{1}{4}F_{ij}(A)F^{ij}(A) + \partial_i A_0 P^i - \frac{1}{2}P^i P_i + \mu\psi^\dagger\psi - \psi^\dagger(a_0 + eA_0)\psi - \frac{1}{2m_{\rm b}}\psi^\dagger\vec{\mathcal{D}}^2\psi. \tag{2.8}$$

Now, we must implement the consitency condition on the primary constraints and find the secondary constraints. From Eqs. (2.6a,c), we find the following secondary constraints:

$$\Phi_1^1 = \left[\Phi_1^0, H_{\rm p}\right] = \frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij}\partial_i a_j + \partial_i p^i + \psi^\dagger\psi \approx 0, \tag{2.9a}$$

$$\Phi_3^1 = \left[\Phi_3^0, H_{\rm p}\right] = \partial_i P^i + e\psi^\dagger\psi \approx 0. \tag{2.9b}$$

Equations (2.9a,b) are the time components of the equations of motion corresponding to a_μ and A_μ, respectively.

Once the consistency condition is imposed on the constraints (2.6b,d,e), the Lagrange multipliers λ_{2i}, $i = 1, 2$, $\lambda_\alpha^\dagger$ and λ_α, $\alpha = 1, 2$, appearing in Eq. (2.7) are determined.

Later on, when the consistency on the constraints $\Phi_1^1 \approx 0$ and $\Phi_3^1 \approx 0$ is imposed, the equations are identically satisfied, so no new constraint exists.

Consequently, in the model there are six bosonic constraints, (2.6a–c) and (2.9), and four fermionic ones, (2.6d,e). It is easy to show that the two bosonic constraints (2.6a,c) are first-class, while the other four bosonic, together with the four fermionic ones, are second-class. Moreover, it can be proven that there are two suitable linear combinations of second-class constraints that give rise to two new first-class constraints. These linear combinations are written as

$$\Sigma_1 = e\Phi_1^1 - \Phi_3^1 = e\partial_i p^i - \partial_i P^i + \frac{e}{4\pi\tilde{\phi}}\varepsilon^{ij}\partial_i a_j \approx 0, \tag{2.10a}$$

$$\Sigma_2 = \psi^\dagger\Omega - \psi\Omega^\dagger - \frac{i}{e}\Phi_3^1 = \psi^\dagger\pi - \psi\pi^\dagger - \frac{i}{e}\partial_i P^i \approx 0. \tag{2.10b}$$

Therefore, two of the second-class constraints can be eliminated and the final set of constraints remains given by

(i) The four bosonic first-class constraints defined by the functions Σ_1, Σ_2, $\Sigma_3 = \Phi_1^0$ and $\Sigma_4 = \Phi_3^0$.

(ii) The two bosonic second-class constraints defined by Φ_2^{0i}, $i = 1, 2$, and the four fermionic second-class constraints defined by $\Omega_\alpha^\dagger$ and Ω_α, $\alpha = 1, 2$.

Now, we must go from the Bose–Fermi brackets to the Dirac brackets $D(F)$ with regard to the matrix F constructed with the Bose–Fermi brackets between the second-class constraints. It is well-known that the $D(F)$ bracket between the variables $R(x)$ and $S(y)$ is defined by

$$[R(x), S(y)]^{D(F)} = [R(x), S(y)] - \int d^2u\, d^2v [R(x), \Gamma_I(u)] F^{-1}_{IJ}(\vec{u}, \vec{v}) [\Gamma_J(v), S(y)], \quad (2.11)$$

where $I, J = 1, \ldots, 6$ and $\Gamma_1 = \Phi_2^{01}$, $\Gamma_2 = \Phi_2^{02}$, $\Gamma_3 = \Omega_1^\dagger$, $\Gamma_4 = \Omega_2^\dagger$, $\Gamma_5 = \Omega_1$, and $\Gamma_6 = \Omega_2$ are the second-class constraints.

In Eq. (2.11), the matrix F^{-1} is the inverse of the matrix F with elements $[\Gamma_I, \Gamma_J]$. It is easy to show that the matrix F is written as follows:

$$F = \begin{pmatrix} 0 & -\frac{1}{2\pi\tilde{\phi}} & 0 & 0 & 0 & 0 \\ \frac{1}{2\pi\tilde{\phi}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 \end{pmatrix} \delta(\vec{x} - \vec{y}) \quad (2.12)$$

and the inverse reads

$$F^{-1} = \begin{pmatrix} 0 & 2\pi\tilde{\phi} & 0 & 0 & 0 & 0 \\ -2\pi\tilde{\phi} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & 0 \end{pmatrix} \delta(\vec{x} - \vec{y}). \quad (2.13)$$

Now, the extended Hamiltonian H_e, generator of the time evolutions of generic functionals, remains defined as follows:

$$H_e = \int d^2x (\mathcal{H}_c + \rho^a \Sigma_a) - \int d^2x\, d^2y\, \Gamma_I(x) F^{-1}_{IJ}(\vec{x}, \vec{y}) [\Gamma_J(y), H_c]. \quad (2.14)$$

In Eq. (2.14), ρ^a, $a = 1, \ldots, 4$, are four arbitrary parameters and the four first-class constraints associated correspond to the symmetries of the gauge group $U(1) \times U(1)$.

Once we impose the $D(F)$ brackets we must take the second-class constraints as strongly equal to zero equations. So, the second term on the right-hand side of Eq. (2.14) vanishes and then the extended Hamiltonian writes

$$H_e = \int d^2x (\mathcal{H}_c + \rho^a \Sigma_a). \quad (2.15)$$

Furthermore, the following fields are determined:

$$p^i = \frac{1}{4\pi\tilde{\phi}} \varepsilon^{ij} a_{j,} \quad (2.16a)$$

$$\pi_\alpha^\dagger = -i\frac{\tau+1}{2}\psi_\alpha^\dagger, \tag{2.16b}$$

$$\pi_\alpha = i\frac{\tau-1}{2}\psi_\alpha. \tag{2.16c}$$

So, from Eq. (2.11), we find the following $D(F)$ brackets:
field–field:

$$[a_1(x), a_2(y)]_-^{D(F)} = 2\pi\tilde{\phi}\delta(\vec{x}-\vec{y}), \tag{2.17a}$$

$$[\psi_\alpha^\dagger(x), \psi_\beta(y)]_+^{D(F)} = -i\delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{2.17b}$$

field–momentum:

$$[a_0(x), p^0(y)]_-^{D(F)} = \delta(\vec{x}-\vec{y}), \tag{2.17c}$$

$$[A_\mu(x), P^\nu(y)]_-^{D(F)} = \delta_\mu^\nu\delta(\vec{x}-\vec{y}), \tag{2.17d}$$

with all other brackets vanishing.

Now, we must calculate the final Dirac brackets. For this purpose, we must search for admissible gauge-fixing conditions $\Theta_a \approx 0$, $a = 1, \ldots, 4$, each one of them corresponding to each first-class constraint.

Moreover, these subsidiary conditions must verify that $\det[\Delta_A, \Delta_B] \not\approx 0$, where $A, B = 1, \ldots, 8$ and $\Delta_a = \Sigma_a$, $\Delta_{4+a} = \Theta_a$, $a = 1, \ldots, 4$, and must be compatible with the equations of motion. Of course, the above requirements do not determine uniquely the functions Θ_a and so the matter is to find a suitable set of gauge-fixing conditions.

Let us assume the following simple expressions for the gauge-fixing conditions:

$$\Theta_1 = \partial^i a_i \approx 0, \tag{2.18a}$$

$$\Theta_2 = \partial^i A_i \approx 0, \tag{2.18b}$$

$$\Theta_3 = a_0 \approx 0, \tag{2.18c}$$

$$\Theta_4 = \nabla^2 A_0 - \partial_i P^i \approx 0. \tag{2.18d}$$

The Dirac bracket between the variables $R(x)$ and $S(y)$ is written as

$$[R(x), S(y)]^D = [R(x), S(y)]^{D(F)} - \int d^2u\, d^2v [R(x), \Delta_A(u)]^{D(F)} \times G_{AB}^{-1}(\vec{u}, \vec{v})[\Delta_B(v), S(y)]^{D(F)}. \tag{2.19}$$

In Eq. (2.19), the matrix G^{-1} is the inverse of the matrix G whose elements are $[\Delta_A, \Delta_B]$, $A, B = 1, \ldots, 8$.

It is easy to see that the matrix G is given by

$$G = \begin{pmatrix} 0 & 0 & 0 & 0 & e\nabla^2 & -\nabla^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{e}\nabla^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\nabla^2 \\ -e\nabla^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \nabla^2 & \frac{i}{e}\nabla^2 & 0 & 0 & 0 & 0 & 0 & \nabla^2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \nabla^2 & 0 & -\nabla^2 & 0 & 0 \end{pmatrix} \delta(\vec{x} - \vec{y}). \quad (2.20)$$

The determinant of G holds

$$\det G = -[\nabla^2]^6 \delta(\vec{x} - \vec{y}) \not\approx 0 \quad (2.21)$$

and its inverse matrix is written as

$$G^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & -e^{-1}u & 0 & 0 & 0 \\ 0 & 0 & 0 & -ieu & -iu & -ieu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & v & 0 \\ 0 & ieu & 0 & 0 & 0 & 0 & 0 & u \\ e^{-1}u & iu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & ieu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -v & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -u & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (2.22)$$

where $u = (4\pi|\vec{x} - \vec{y}|)^{-1}$ and $v = \delta(\vec{x} - \vec{y})$.

Once we impose the Dirac brackets we must take the first-class constraints and the gauge-fixing conditions as strongly equal to zero equations. So, the following fields are determined:

$$p^0 = 0, \quad (2.23a)$$

$$P^0 = 0, \quad (2.23b)$$

$$a_0 = 0, \quad (2.23c)$$

$$A_0(x) = \frac{1}{4\pi} \int d^2y \frac{\partial_i P^i(y)}{|\vec{x} - \vec{y}|}. \quad (2.23d)$$

This way, from Eq. (2.19), we obtain the following Dirac brackets:
field–field:

$$[a_1(x), a_2(y)]_-^D = 2\pi\tilde{\phi}\delta(\vec{x} - \vec{y}), \quad (2.24a)$$

$$[\psi_\alpha^\dagger(x), \psi_\beta(y)]_+^D = -i\delta_{\alpha\beta}\delta(\vec{x} - \vec{y}), \quad (2.24b)$$

field–momentum:

$$[A_i(x), P^j(y)]_-^D = \delta_i^j \delta(\vec{x} - \vec{y}) - \frac{1}{4\pi} \partial_i(x) \partial^j(x) \frac{1}{|\vec{x} - \vec{y}|}, \tag{2.24c}$$

with all other brackets vanishing.

So, the dynamics of the classical model is then completely specified.

Finally, the canonical quantization is realized by replacing the Dirac brackets between field variables in the (anti) commutators between field operators according to the rule

$$[O_1(x), O_2(y)]^D \to -i[\hat{O}_1 \hat{O}_2 - (-1)^{|O_1||O_2|} \hat{O}_2 \hat{O}_1], \tag{2.25}$$

where $|O_i| = 0(1)$ when O_i is bosonic (fermionic), $i = 1, 2$.

Consequently, the four first-class constraints and the corresponding four gauge-fixing conditions that we have determined restrict the phase space variables to the physical ones, and so the true Hilbert space is obtained.

Furthermore, we note that the quantization of a nonrelativistic CB system interacting with the electromagnetic field can be treated similarly. In this case, the four second-class fermionic constraints (2.6d,e) turn into second-class bosonic ones and no other change in the structure of the constraints occurs. The fermionic brackets (2.5c,d), (2.17b), and (2.24b) turn into bosonic brackets. So, after that Eq. (2.25) is imposed, the Dirac brackets (2.24b) become commutators.

3. PATH INTEGRAL QUANTIZATION AND FEYNMAN RULES

We develop the Feynman path integral quantization method according to the Faddeev–Senjanovic (FS) formalism (Faddeev, 1970; Senjanovic, 1976) used when the system has first- and second-class constraints. So, we assume that the partition function for the present gauge model is written as follows:

$$Z = \int \mathbb{D}a_\mu \mathbb{D}p^\mu \mathbb{D}A_\nu \mathbb{D}P^\nu \mathbb{D}\psi_\alpha \mathbb{D}\pi_\alpha^\dagger \mathbb{D}\psi_\beta^\dagger \mathbb{D}\pi_\beta \delta(\Delta_A)(\det G)^{1/2} \delta(\Gamma_I)(\det F)^{1/2}$$
$$\times \exp\left\{ i \int d^3x [\dot{a}_\mu p^\mu + \dot{A}_\mu P^\mu + \dot{\psi}\pi^\dagger + \dot{\psi}^\dagger \pi - \mathcal{H}_e] \right\}, \tag{3.1}$$

where the Hamiltonian density $\mathcal{H}_e$ was given in Eq. (2.14).

The determinant of the matrix (2.12) holds

$$\det F = \frac{1}{4\pi^2 \tilde{\phi}^2} \delta(\vec{x} - \vec{y}). \tag{3.2}$$

As it does not depend either on the field variables or on the corresponding canonically conjugate momenta, thus $(\det F)^{1/2}$ is included in the path integral normalization factor. Exactly the same occurs with the other determinant appearing in Eq. (3.1) (see Eq. (2.21)).

In Eq. (3.1), by using the delta functions $\delta(\Delta_3)$, $\delta(\Delta_4)$, $\delta(\Delta_7)$, $\delta(\Gamma_3)$, $\delta(\Gamma_4)$, $\delta(\Gamma_5)$, and $\delta(\Gamma_6)$, the path integrals over the fields p^0, P^0, a_0, $\pi_1^\dagger$, $\pi_2^\dagger$, π_1, and π_2, respectively, are immediately performed.

Moreover, in Eq. (3.1), the delta function $\delta(\Delta_8)$ can be written as $\delta(\Delta_8) = \delta(A_0 - f)$, where $f(x) = \frac{1}{4\pi} \int d^2y \, \frac{\partial_i P^i(y)}{|\vec{x}-\vec{y}|}$, and also the path integral on A_0 can be made.

On the other hand, we use the Fourier integral representation $\delta(\Delta_k) = \int \mathbb{D}\Lambda^k \exp\left(i \int d^3x \; \Lambda^k \Delta_k\right)$, $k = 1, 2$.

Consequently, the expression (3.1) of the partition function can be written as

$$Z = \int \mathbb{D}a_i \mathbb{D}p^i \mathbb{D}A_j \mathbb{D}P^j \mathbb{D}\psi_\alpha \mathbb{D}\psi_\beta^\dagger \mathbb{D}\Lambda^k \delta\left(\partial^l a_l\right) \delta\left(\partial^m A_m\right) \delta\left(\Gamma_1\right) \delta(\Gamma_2) \times \exp\left(i \int d^3x \, \mathcal{L}'\right), \tag{3.3}$$

where

$$\mathcal{L}' = \dot{a}_i p^i + \dot{A}_i P^i + \frac{i}{2}[(\tau - 1)\dot{\psi}^\dagger \psi - (\tau + 1)\dot{\psi}\psi^\dagger] - \mathcal{H}', \tag{3.4}$$

with

$$\mathcal{H}' = \mathcal{H}'_{\mathrm{e}} - \Lambda^k \Delta_k, \tag{3.5}$$

where $k = 1, 2$ and $\mathcal{H}'_{\mathrm{e}}$ is the original $\mathcal{H}_{\mathrm{e}}$ subject to the integrations that we have just done.

Looking at Eq. (3.5), due to the arbitrariness of the multipliers Λ^k, it is possible to rescale the corresponding integration variables in such a way as to have $\mathcal{H}' = \mathcal{H}_{\mathrm{c}}$ (Sundermeyer, 1982).

In Eq. (3.3), by using the delta functions $\delta(\Gamma_1)$ and $\delta(\Gamma_2)$, the path integrals over the fields p^1 and p^2, respectively, are carried out.

The integrations over the variables P^j, which are Gaussian integrals, can also be performed and so the partition function takes the form

$$Z = \int \mathbb{D}a_\mu \mathbb{D}A_\nu \mathbb{D}\psi_\alpha \mathbb{D}\psi_\beta^\dagger \delta\left(\partial^l a_l\right) \delta\left(\partial^m A_m\right) \exp\left(i \int d^3x \, \mathcal{L}\right), \tag{3.6}$$

where $\mathcal{L}$ is the original Lagrangian density written in Eq. (2.1).

Finally, by using the Faddeev–Popov trick to go over a general covariant gauge, we write the gauge-fixing conditions in the form $\partial^\mu a_\mu(x) - c_a(x) = 0$ and $\partial^\mu A_\mu(x) - c_A(x) = 0$. By considering the first of these conditions, we write $\delta[\partial^\mu a_\mu(x) - c_a(x)] = \int \mathbb{D}c_a(x) \exp\{i \frac{\lambda_a}{2} \int d^3x [\partial^\mu a_\mu(x)]^2\}$, with a Gaussian weight independent of $c_a(x)$. Therefore, the partition function does not depend on $c_a(x)$ and the integration over this quantity can be carried out. So, in the path integral we write $\exp\{i\frac{\lambda_a}{2}\int d^3x[\partial^\mu a_\mu(x)]^2\}$ instead of $\delta[\partial^\mu a_\mu(x) - c_a(x)]$. Analogously for the second condition.

This way, Eq. (3.6) takes the final form

$$Z = \int \mathbb{D}a_\mu \mathbb{D}A_\nu \mathbb{D}\psi_\alpha \mathbb{D}\psi^\dagger_\beta \exp\left(i \int d^3x\, \mathcal{L}_{\text{eff}}\right), \tag{3.7}$$

where the functional $\mathcal{L}_{\text{eff}}$ is given by

$$\mathcal{L}_{\text{eff}} = \mathcal{L} + \mathcal{L}_{\text{fix}}, \tag{3.8}$$

with

$$\mathcal{L}_{\text{fix}} = \frac{\lambda_a}{2}(\partial^\mu a_\mu)^2 + \frac{\lambda_A}{2}(\partial^\mu A_\mu)^2. \tag{3.9}$$

So, a profitable form for the partition function was obtained by expressing the quantization of the system under consideration in terms of a path integral for the independent dynamical fields of the model, a_μ, A_μ, ψ_α, and $\psi^\dagger_\alpha$, which take place. Next, we can use the diagrammatic technique in the framework of the perturbative theory.

It is straightforward to go from the path integral (3.7) to the Feynman rules for propagators and vertices ('t Hooft and Velman, 1973). So, the quadratic part of the Lagrangian density $\mathcal{L}_{\text{eff}}$ is recognized as representing the propagators and the remaining pieces as representing the vertices. Consequently, $\mathcal{L}_{\text{eff}}$ stands for the effective Lagrangian density of a CF system coupled to the electromagnetic field and it can be partitioned

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}\,(a_\mu) + \mathcal{L}_{\text{eff}}\,(A_\mu) + \mathcal{L}_{\text{eff}}\,(\psi, \psi^\dagger) + \mathcal{L}^{\text{int}}_{\text{eff}}\,(a_\mu, A_\mu, \psi, \psi^\dagger). \tag{3.10}$$

We have named

$$\mathcal{L}_{\text{eff}}\,(a_\mu) = \frac{1}{2}a_\mu (d^{-1})^{\mu\nu} a_\nu, \tag{3.11a}$$

$$\mathcal{L}_{\text{eff}}\,(A_\mu) = \frac{1}{2}A_\mu (D^{-1})^{\mu\nu} A_\nu, \tag{3.11b}$$

$$\mathcal{L}_{\text{eff}}\,(\psi, \psi^\dagger) = \psi^\dagger G^{-1} \psi, \tag{3.11c}$$

$$\begin{aligned}\mathcal{L}^{\text{int}}_{\text{eff}}\,(a_\mu, A_\mu, \psi, \psi^\dagger) &= \psi^\dagger V^1_\mu a^\mu \psi + \psi^\dagger V^2_\mu A^\mu \psi + \psi^\dagger a_\mu W^{\mu\nu}_1 a_\nu \psi \\ &\quad + \psi^\dagger A_\mu W^{\mu\nu}_2 A_\nu \psi + \psi^\dagger a_\mu W^{\mu\nu}_3 A_\nu \psi. \end{aligned} \tag{3.11d}$$

In Eq. (3.11a), the 3×3 matrix (d^{-1}) is the inverse of the propagator matrix of the CS field a_μ. Analogously, in Eq. (3.11b), the 3×3 matrix (D^{-1}) is the inverse of the propagator matrix of the electromagnetic field A_μ. These matrices are Hermitian and nondegenerate. So, the propagators $d_{\mu\nu}(k)$ and $D_{\mu\nu}(k)$, in the momentum space, can be evaluated and read

$$d_{\mu\nu}(k) = \frac{1}{\lambda_a}\frac{k_\mu k_\nu}{k^4} + 2i\pi\tilde{\phi}\varepsilon_{\mu\nu\rho}\frac{k^\rho}{k^2}, \tag{3.12a}$$

$$D_{\mu\nu}(k) = -g_{\mu\nu}\frac{1}{k^2} + \left(1 + \frac{1}{\lambda_A}\right)\frac{k_\mu k_\nu}{k^4}, \tag{3.12b}$$

where $k^2 = k_\mu k^\mu$.

In Eq. (3.11c), G is the nonrelativistic propagator of the matter field. In the momentum space, it is given by

$$G(\vec{p}, E) = \left(E - \mu - \frac{\vec{p}^2}{2m_\mathrm{b}}\right)^{-1}, \tag{3.13}$$

where E is the particle energy, $\vec{p}$ is its ordinary momentum, and $\vec{p}^2 = p_1^2 + p_2^2$.

In Eq. (3.11d), the 3-vectors $V^n = (V^n_\mu)$, $n = 1, 2$, give the 3-point vertices of the model and read

$$V^1 = V, \tag{3.14a}$$

$$V^2 = eV, \tag{3.14b}$$

where

$$V = \left(1, \frac{1}{m_\mathrm{b}}q_i\right), \tag{3.15}$$

with $i = 1, 2$.

Finally, in Eq. (3.11d), the 3×3 matrices $W_m = (W_m^{\mu\nu})$, $m = 1, 2, 3$, give the 4-point vertices and are written as

$$W_1 = -\frac{1}{2m_\mathrm{b}}W, \tag{3.16a}$$

$$W_2 = -\frac{e^2}{2m_\mathrm{b}}W, \tag{3.16b}$$

$$W_3 = -\frac{e}{m_\mathrm{b}}W, \tag{3.16c}$$

where

$$W = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.17}$$

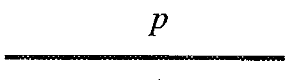
Next, the Feynman rules for propagators and vertices can be written:

(i) *Propagators.* We associate with the propagators of the gauge fields a_μ and A_μ a wavy thick line and a wavy thin line connecting two generic points, respectively,

$$\equiv d_{\mu\nu}(k),$$

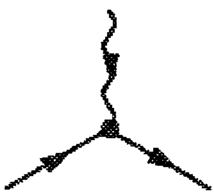
$$\equiv D_{\mu\nu}(k)$$

and with a straight line the usual nonrelativistic propagator of the fermionic matter field ψ



$$\equiv G(\vec{p}, E).$$

(ii) *Vertices.* So, the 3-point vertices of the model are

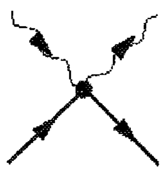


$$\equiv V_\mu^1,$$

$$\equiv V_\mu^2$$

and the 4-point vertices are

$$\equiv W_1^{\mu\nu},$$



$$\equiv W_2^{\mu\nu},$$

$$\equiv W_3^{\mu\nu}.$$

Here, we have not included the factor $(2\pi)^3$ and momentum conservation delta functions. They are to be understood. Furthermore, as is usual, we have to take into account a minus sign for every closed fermion loop and another minus sign for diagrams related through the exchange of two fermion lines, internal or external. A combinatorial factor correcting for double counting in case that identical particles occur must also be taken into account.

We do not treat the regularization and renormalization problem of this model here. However, by considering the expressions of the propagators and vertices and taking into account the above Feynman rules, complete information about the perturbative behavior could be obtained. It can be seen that this gauge model belongs to the class of theories with only a finite number of divergent diagrams. So, the regularization and renormalization problem is reduced to the one corresponding to a superrenormalizable theory and it can be solved by the usual methods.

4. SIMPLIFIED MODEL

Now, we consider the following singular Lagrangian density:

$$\mathcal{L} = i\frac{\tau+1}{2}\psi^\dagger\partial_0\psi + i\frac{\tau-1}{2}\partial_0\psi^\dagger\psi - \mu\psi^\dagger\psi + \psi^\dagger a_0\psi + \frac{1}{2m_\mathrm{b}}\psi^\dagger\vec{\mathcal{D}}^2\psi$$
$$+ \frac{1}{2\pi\tilde{\phi}}\varepsilon^{ij}a_0\partial_i a_j, \tag{4.1}$$

obtained by removing few terms from our starting one (see Eq. (2.1)).

The Lagrangian density (4.1) is essentially that considered by Halperin *et al.* (1993).

The momenta p^μ, P^i, $\pi^\dagger_\alpha$, and π_α, canonically conjugate to the independent field variables a_μ, A_i, ψ_α, and $\psi^\dagger_\alpha$, respectively, read

$$p^0 = 0, \tag{4.2a}$$

$$p^i = 0, \tag{4.2b}$$

$$P^i = 0, \tag{4.2c}$$

$$\pi^\dagger_\alpha = \frac{\partial\mathcal{L}}{\partial\dot{\psi}_\alpha} = -i\frac{\tau+1}{2}\psi^\dagger_\alpha, \tag{4.2d}$$

$$\pi_\alpha = \frac{\partial\mathcal{L}}{\partial\dot{\psi}^\dagger_\alpha} = i\frac{\tau-1}{2}\psi_\alpha, \tag{4.2e}$$

where $i = 1, 2$ and $\alpha = 1, 2$.

The Bose–Fermi brackets are given by Eqs. (2.5a,c,d) and the spatial part of Eq. (2.5b).

From Eqs. (4.2), we obtain the primary constraints

$$\Phi^0_1 = p^0 \approx 0, \tag{4.3a}$$

$$\Phi^{0i}_2 = p^i \approx 0, \tag{4.3b}$$

$$\Phi_3^{0i} = P^i \approx 0, \tag{4.3c}$$

$$\Omega_\alpha^\dagger = \pi_\alpha^\dagger + i\frac{\tau+1}{2}\psi_\alpha^\dagger \approx 0, \tag{4.3d}$$

$$\Omega_\alpha = \pi_\alpha - i\frac{\tau-1}{2}\psi_\alpha \approx 0, \tag{4.3e}$$

with $i = 1, 2$ and $\alpha = 1, 2$.

In this case, the primary Hamiltonian density reads

$$\mathcal{H}_\mathrm{p} = \mathcal{H}_\mathrm{c} + \lambda_1\Phi_1^0 + \lambda_{2i}\Phi_2^{0i} + \lambda_{3i}\Phi_3^{0i} + \lambda_\alpha^\dagger\Omega_\alpha + \Omega_\alpha^\dagger\lambda_\alpha, \tag{4.4}$$

where λ_1, λ_{2i}, and λ_{3i}, $i = 1, 2$, are bosonic Lagrange multipliers and $\lambda_\alpha^\dagger$ and λ_α, $\alpha = 1, 2$, are fermionic ones.

In Eq. (4.4), $\mathcal{H}_\mathrm{c} = \dot{a}_\mu p^\mu + \dot{A}_i P^i + \dot{\psi}^\dagger\pi + \dot{\psi}\pi^\dagger - \mathcal{L}$ and by using Eqs. (4.2) we obtain

$$\mathcal{H}_\mathrm{c} = \mu\psi^\dagger\psi - \psi^\dagger a_0\psi - \frac{1}{2m_\mathrm{b}}\psi^\dagger\vec{\mathcal{D}}^2\psi - \frac{1}{2\pi\tilde{\phi}}\varepsilon^{ij}a_0\partial_i a_j. \tag{4.5}$$

We implement the consistency condition on the bosonic constraints (4.3a,b,c,), and we find the following secondary constraints:

$$\Phi_1^1 = \frac{1}{2\pi\tilde{\phi}}\varepsilon^{ij}\partial_i a_j + \psi^\dagger\psi \approx 0, \tag{4.6a}$$

$$\Phi_2^{1i} = \varepsilon^{ik}\partial_k a_0 \approx 0, \tag{4.6b}$$

$$\Phi_3^{1i} = \psi^\dagger(i\partial_i + a_i + eA_i)\psi \approx 0, \tag{4.6c}$$

where $i = 1, 2$.

Equation (4.6a) is the time component of the equations of motion corresponding to a_μ.

Once the consistency condition is imposed on the fermionic constraints (4.3d,e), the Lagrange multipliers $\lambda_\alpha^\dagger$ and λ_α, $\alpha = 1, 2$, appearing in Eq. (4.4) are determined. Later on, when the consistency on the constraints (4.6) is imposed, the Lagrange multipliers λ_1, λ_{2i}, and λ_{3i}, $i = 1, 2$, present in Eq. (4.4) are determined.

We find that every constraint is a second-class one.

Summarizing, in the model under consideration there are fourteen second-class constraints, ten bosonic, (4.3a–c) and (4.6), and four fermionic ones (4.3d,e).

5. BRST FORMALISM

We are going to construct the BRST formalism for the constrained Hamiltonian system under consideration (Becchi *et al.*, 1976; Fradkin and

Fradkina, 1978; Fradkin and Vilkovisky, 1975; Henneaux, 1985; Marnelius, 1981; Sundermeyer, 1982; Tyupin, unpublished).

As was shown above, the Hamiltonian system before imposing the gauge-fixing conditions is defined by the four first-class constraints $\Sigma_a \approx 0$, $a = 1, \ldots, 4$, given in Eqs. (2.10) and (2.6a,c), respectively, the Hamiltonian given in Eq. (2.15) and the $D(F)$ brackets for the dynamical variables given in Eqs. (2.17). The following brackets are all $D(F)$ brackets. So, from now on we will write the $D(F)$ brackets without the superscription "$D(F)$".

At this stage, it is convenient to note that the Hamiltonian density $\mathcal{H}_{\mathrm{c}}$, given in Eq. (2.8), can be partitioned in the following way:

$$\mathcal{H}_{\mathrm{c}} = \mathcal{H}_0 - a_0\left(\frac{1}{e}\Sigma_1 + i\Sigma_2\right) - ieA_0\Sigma_2. \tag{5.1}$$

We can write

$$[\Sigma_a(x), \Sigma_b(y)]_- = C^c_{ab}\Sigma_c(x)\delta(\vec{x} - \vec{y}), \tag{5.2a}$$

$$[H_0, \Sigma_a(x)]_- = D^b_a\Sigma_b(x), \tag{5.2b}$$

with $a, b, c = 1, \ldots, 4$ and $H_0 = \int d^2x\mathcal{H}_0$, where $\mathcal{H}_0$ is the particular Hamiltonian density from Eq. (5.1).

In Eq. (5.2a), let us note that for the constrained Hamiltonian system under consideration all the coefficients C^c_{ab} vanish. This is a consequence that the present model is an Abelian one.

Furthermore, in Eq. (5.2b), all the coefficients D^b_a vanish for the Hamiltonian chosen H_0. This is always possible to make in any usual CS theory (Henneaux, 1985). For instance, one case in which this choice is not feasible is when terms with higher derivatives are added to the Lagrangian (Foussats *et al.*, 1995).

Moreover, owing to the arbitrariness of the Lagrange multipliers, the Hamiltonian density appearing in Eq. (2.15) can also be written as follows:

$$\mathcal{H}_{\mathrm{e}} = \mathcal{H}_0 - \rho^a\Sigma_a, \tag{5.3}$$

where $a = 1, \ldots, 4$, with the relation between the Hamiltonian densities $\mathcal{H}_0$ and $\mathcal{H}_{\mathrm{c}}$ given in Eq. (5.1).

As is well-known, in the BRST formalism it is convenient to treat the Lagrange multipliers ρ_a (defined in Eq. (5.3)) on the same footing as the remaining dynamical variables and to associate with them an equal number of canonically conjugate momenta ξ^a, such as

$$[\rho_a(x), \xi^b(y)]_- = \delta^b_a\delta(\vec{x} - \vec{y}). \tag{5.4}$$

With the purpose of not changing the dynamical structure of the theory, classically these momenta are constrained to vanish. Precisely, the first-class constraints $\xi_a = 0$ generate the gauge transformations $\rho_a \rightarrow \rho_a + u_a$ of the multipliers, making evident their arbitrariness.

Consequently, from now on our set of dynamical variables will be

$$A_\Sigma = (a_\mu, A_\mu, \psi_\alpha, \psi^\dagger_\alpha, \rho_a), \tag{5.5}$$

where the compound index Σ we have used runs over the components of the field variables.

The set of the canonically conjugate momenta corresponding to the field variables is written as

$$P^\Sigma = (p^\mu, P^\mu, \pi^\dagger_\alpha, \pi_\alpha, \xi^a) \tag{5.6}$$

and the new set of first-class constraints is defined by the functions

$$\Xi_A = (\Sigma_a, \xi_a), \tag{5.7}$$

where $A = 1, \ldots, 8$.

Therefore, Eqs. (5.2) take the form

$$[\Xi_A(x), \Xi_B(y)]_- = C^C_{AB}\Xi_C(x)\delta(\vec{x} - \vec{y}), \tag{5.8a}$$

$$[H_0, \Xi_A(x)]_- = D^B_A\Xi_B(x), \tag{5.8b}$$

where all the coefficients C^C_{AB} and D^B_A vanish.

Now, we must introduce the BRST-invariant Hamiltonian density $\mathcal{H}_1$ by considering the fermionic ghost fields (Majorana spinors) Q_A and their canonically conjugate momenta P^A satisfying

$$[\mathsf{Q}_A(x), \mathsf{P}^B(y)]_+ = \delta^B_A\delta(\vec{x} - \vec{y}). \tag{5.9}$$

This Hamiltonian density is written as

$$\mathcal{H}_1 = \mathcal{H}_0 + \mathsf{P}_B D^B_A \mathsf{Q}^A = \mathcal{H}_0. \tag{5.10}$$

Next, we must find the BRST-invariant gauge-fixed Hamiltonian density $\mathcal{H}_\chi$; it is given by

$$\mathcal{H}_\chi(x) = \mathcal{H}_1(x) - \int d^2y[\chi(x), Q(y)]_+ = \mathcal{H}_0(x) - \int d^2y[\chi(x), Q(y)]_+, \tag{5.11}$$

where $\chi = \mathsf{P}_A\Upsilon^A$ is the gauge-fixing variable, Υ^B being the functions that define the gauge-fixing conditions given by the set of quantities

$$\Upsilon^A = -(\rho^a, \Theta^a). \tag{5.12}$$

In Eq. (5.11), Q is the BRST generator given by the well-known expression

$$Q = \Xi_A \mathsf{Q}^A + \frac{1}{2}\mathsf{P}_C C^C_{AB}\mathsf{Q}^A\mathsf{Q}^B = \Xi_A \mathsf{Q}^A. \tag{5.13}$$

As the constraints defined by the functions (5.7) can be partitioned into two subsets, we assume that the ghosts are considered in such a way that

$$\mathsf{Q}_A = (\mathsf{q}_a, \mathsf{p}_a), \tag{5.14a}$$

$$\mathsf{P}^A = (\mathsf{p}^{\dagger a}, \mathsf{q}^{\dagger a}), \tag{5.14b}$$

inserting the antighosts. So, we are working in the mixed representation in which both the ghosts and antighosts are diagonal.

Therefore, the following canonical brackets hold:

$$[\mathsf{q}_a(x), \mathsf{p}^{\dagger b}(y)]_+ = \delta^b_a \delta(\vec{x} - \vec{y}), \tag{5.15a}$$

$$[\mathsf{p}_a(x), \mathsf{q}^{\dagger b}(y)]_+ = \delta^b_a \delta(\vec{x} - \vec{y}). \tag{5.15b}$$

Like this, we obtain the following expression for $\mathcal{H}_\chi$:

$$\mathcal{H}_\chi(x) = \mathcal{H}_0(x) + \mathsf{p}^\dagger_a(x)\mathsf{p}^a(x) + \Sigma_a(x)\rho^a(x) + \xi_a(x)\Theta^a(x) + \mathsf{q}^\dagger_a(x)\int d^2y[\Theta^a(x), \Sigma_b(y)]_- \mathsf{q}^b(y). \tag{5.16}$$

When an integration in the last term of Eq. (5.16) is performed and since $[\Theta^a(x), \Sigma_b(y)]_- = f^a_b \nabla^2 \delta(\vec{x} - \vec{y}) + g^a_b \delta(\vec{x} - \vec{y})$, where f^a_b and g^a_b are the elements of the matrices

$$f = \begin{pmatrix} -e & 0 & 0 & 0 \\ 1 & \frac{i}{e} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{5.17a}$$

$$g = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{5.17b}$$

respectively, this term reads $f^a_b \mathsf{q}^\dagger_a(x)\nabla^2\mathsf{q}^b(x) + \mathsf{q}^\dagger_3(x)\mathsf{q}^3(x)$.

Consequently, the BRST Lagrangian density $\mathcal{L}_\chi$ is given by

$$\mathcal{L}_\chi = \dot{A}_\Sigma P^\Sigma + \mathsf{P}^A \dot{\mathsf{Q}}_A - \mathcal{H}_\chi. \tag{5.18}$$

When the constrained system has first- and second-class constraints, as in the present case, the partition function in the BRST formalism is written using the following path integral (Fradkin and Fradkina, 1978; Henneaux, 1985):

$$Z_\chi = \int \mathbb{D}A_\Sigma \mathbb{D}P^\Sigma \mathbb{D}\mathsf{Q}_A \mathbb{D}\mathsf{P}^A \delta(\Gamma_I)(\det F)^{1/2} \exp\left(i \int d^3x \mathcal{L}_\chi\right), \tag{5.19}$$

where det F is given by Eq. (3.2).

It is easy to prove that this last expression for the partition function is equivalent to that of Eq. (3.1) in the FS form, as follows:

The path integral over the variables p_a and $\mathsf{p}^{\dagger a}$ is Gaussian and it is easily performed.

The next task is to pass to a nonrelativistic gauge (see Sundermeyer, 1982; Henneaux, 1985); so, we carry out the following replacement $\Theta^a \to \varepsilon^{-1}\Theta^a$, make a scale change of the integration variables $\xi^a \to \varepsilon\xi^a$ and $\mathsf{q}_a^\dagger \to \varepsilon \mathsf{q}_a^\dagger$, and pass to the limit $\varepsilon \to 0$ in the partition function (this is possible because of the Fradkin–Vilkovisky theorem), obtaining

$$Z_\chi = \int \mathbb{D}a_\mu \mathbb{D}p^\mu \mathbb{D}A_\nu \mathbb{D}P^\nu \mathbb{D}\psi_\alpha \mathbb{D}\pi_\alpha^\dagger \mathbb{D}\psi_\beta^\dagger \mathbb{D}\pi_\beta \mathbb{D}\rho_a \mathbb{D}\xi^a \mathbb{D}\mathsf{q}_a \mathbb{D}\mathsf{q}^{\dagger b}$$

$$\times\, \delta(\Gamma_I)(\det F)^{1/2} \exp\left(i \int d^3x\, \mathcal{L}'_\chi\right), \tag{5.20}$$

where

$$\mathcal{L}'_\chi = \dot{a}_\mu p^\mu + \dot{A}_\mu P^\mu + \dot{\psi}\pi^\dagger + \dot{\psi}^\dagger \pi - \mathcal{H}_0 - \Sigma^a \rho_a - \xi^a \Theta_a - \mathsf{q}_a^\dagger [\Theta^a, \Sigma_b]_- \mathsf{q}^b. \tag{5.21}$$

The integrations over the path integral variables ρ_a and ξ^a are elementary and, formally, the integration of the last term of Eq. (5.21) is given by

$$\int \mathbb{D}\mathsf{q}_a \mathbb{D}\mathsf{q}^{\dagger b} \exp\left[-i \int d^3x \mathsf{q}_a^\dagger(x) \int d^2y [\Theta^a(x), \Sigma_b(y)]_- \mathsf{q}^b(y)\right]$$

$$= -\frac{i}{2\pi}(\det G)^{1/2}. \tag{5.22}$$

So, the final outcome coincides exactly with Eq. (3.1), obtained following the FS procedure. Therefore, we conclude that both methods give the same basic results and, this way, they can be considered as alternating ones.

6. CONCLUSIONS AND OUTLOOK

Starting from a classical nonrelativistic $U(1) \times U(1)$ gauge model for composite particles interacting with the electromagnetic field in $(2 + 1)$ dimensions, the canonical quantization has been presented. This has been done for the CF case. The model under consideration was analyzed in the framework of the Dirac Hamiltonian formalism.

Later on, by going over the path integral quantization method, the Feynman rules of the model were established. The model has five vertices, two 3-point and three 4-point vertices. So, using the perturbative theory as is usual, it would be possible to obtain information about the regularization and renormalization of the model.

Next, we have analyzed a simplified version of the starting model similar to one used within the framework of condensed matter.

In the last section, the BRST formalism of the gauge model was given. The partition function obtained from this formalism is equivalent to that obtained by the FS method, as must be expected.

In a future paper, using the perturbative theory we are going to analyze the diagrammatic structure at least at one loop of the model.

Furthermore, we will consider a more general $U(1) \times U(1)$ gauge model for composite particles by adding the following terms to the Lagrangian density:

(i) An interaction term between the CS and electromagnetic fields. Therefore in this case a suitable mixed boson propagator associated with these fields, preserving the gauge invariance of the model, will have to be defined.
(ii) Several types of terms with the purpose of improving the infrared and ultraviolet behaviors of this propagator, to render the model less divergent.

We will compare the obtained results with the corresponding ones to other theories (Shankar, 1999).

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